

NAG Toolbox for MATLAB

f07vs

1 Purpose

f07vs solves a complex triangular band system of linear equations with multiple right-hand sides, $AX = B$, $A^T X = B$ or $A^H X = B$.

2 Syntax

```
[b, info] = f07vs(uplo, trans, diag, kd, ab, b, 'n', n, 'nrhs_p',
nrhs_p)
```

3 Description

f07vs solves a complex triangular band system of linear equations $AX = B$, $A^T X = B$ or $A^H X = B$.

4 References

Golub G H and Van Loan C F 1996 *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

Higham N J 1989 The accuracy of solutions to triangular systems *SIAM J. Numer. Anal.* **26** 1252–1265

5 Parameters

5.1 Compulsory Input Parameters

1: **uplo** – string

Indicates whether A is upper or lower triangular.

uplo = 'U'

A is upper triangular.

uplo = 'L'

A is lower triangular.

Constraint: **uplo** = 'U' or 'L'.

2: **trans** – string

Indicates the form of the equations

trans = 'N'

The equations are of the form $AX = B$.

trans = 'T'

The equations are of the form $A^T X = B$.

trans = 'C'

The equations are of the form $A^H X = B$.

Constraint: **trans** = 'N', 'T' or 'C'.

3: **diag** – string

Indicates whether A is a nonunit or unit triangular matrix.

diag = 'N'

A is a nonunit triangular matrix.

diag = 'U'

A is a unit triangular matrix; the diagonal elements are not referenced and are assumed to be 1.

Constraint: **diag** = 'N' or 'U'.

4: **kd** – int32 scalar

k_d , the number of superdiagonals of the matrix A if **uplo** = 'U', or the number of subdiagonals if **uplo** = 'L'.

Constraint: **kd** ≥ 0 .

5: **ab(ldab,*)** – complex array

The first dimension of the array **ab** must be at least **kd** + 1

The second dimension of the array must be at least $\max(1, \mathbf{n})$

The n by n triangular band matrix A .

The matrix is stored in rows 1 to $k_d + 1$, more precisely,

if **uplo** = 'U', the elements of the upper triangle of A within the band must be stored with element A_{ij} in **ab**($k_d + 1 + i - j, j$) for $\max(1, j - k_d) \leq i \leq j$;

if **uplo** = 'L', the elements of the lower triangle of A within the band must be stored with element A_{ij} in **ab**($1 + i - j, j$) for $j \leq i \leq \min(n, j + k_d)$.

If **diag** = 'U', the diagonal elements of A are assumed to be 1, and are not referenced.

6: **b(ldb,*)** – complex array

The first dimension of the array **b** must be at least $\max(1, \mathbf{n})$

The second dimension of the array must be at least $\max(1, \mathbf{nrhs_p})$

The n by r right-hand side matrix B .

5.2 Optional Input Parameters

1: **n** – int32 scalar

Default: The second dimension of the array **ab**.

n , the order of the matrix A .

Constraint: **n** ≥ 0 .

2: **nrhs_p** – int32 scalar

Default: The second dimension of the array **b**.

r , the number of right-hand sides.

Constraint: **nrhs_p** ≥ 0 .

5.3 Input Parameters Omitted from the MATLAB Interface

ldab, ldb

5.4 Output Parameters

1: **b(ldb,*)** – complex array

The first dimension of the array **b** must be at least $\max(1, n)$

The second dimension of the array must be at least $\max(1, \text{nrhs_p})$

The n by r solution matrix X .

2: **info** – int32 scalar

info = 0 unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

info = $-i$

If **info** = $-i$, parameter i had an illegal value on entry. The parameters are numbered as follows:

1: **uplo**, 2: **trans**, 3: **diag**, 4: **n**, 5: **kd**, 6: **nrhs_p**, 7: **ab**, 8: **ldab**, 9: **b**, 10: **ldb**, 11: **info**.

It is possible that **info** refers to a parameter that is omitted from the MATLAB interface. This usually indicates that an error in one of the other input parameters has caused an incorrect value to be inferred.

info > 0

If **info** = i , $a(i, i)$ is exactly zero; A is singular and the solution has not been computed.

7 Accuracy

The solutions of triangular systems of equations are usually computed to high accuracy. See Higham 1989.

For each right-hand side vector b , the computed solution x is the exact solution of a perturbed system of equations $(A + E)x = b$, where

$$|E| \leq c(k)\epsilon|A|,$$

$c(k)$ is a modest linear function of k , and ϵ is the *machine precision*.

If \hat{x} is the true solution, then the computed solution x satisfies a forward error bound of the form

$$\frac{\|x - \hat{x}\|_{\infty}}{\|x\|_{\infty}} \leq c(k) \text{cond}(A, x)\epsilon, \quad \text{provided} \quad c(k) \text{cond}(A, x)\epsilon < 1,$$

where $\text{cond}(A, x) = \| |A^{-1}| |A| |x| \|_{\infty} / \|x\|_{\infty}$.

Note that $\text{cond}(A, x) \leq \text{cond}(A) = \| |A^{-1}| |A| \|_{\infty} \leq \kappa_{\infty}(A)$; $\text{cond}(A, x)$ can be much smaller than $\text{cond}(A)$ and it is also possible for $\text{cond}(A^H)$, which is the same as $\text{cond}(A^T)$, to be much larger (or smaller) than $\text{cond}(A)$.

Forward and backward error bounds can be computed by calling f07vv, and an estimate for $\kappa_{\infty}(A)$ can be obtained by calling f07vu with **norm_p** = 'I'.

8 Further Comments

The total number of real floating-point operations is approximately $8nkr$ if $k \ll n$.

The real analogue of this function is f07ve.

9 Example

```
uplo = 'L';
trans = 'N';
diag = 'N';
kd = int32(2);
ab = [complex(-1.94, +4.43), complex(4.12, -4.27), complex(0.43, -2.66),
      complex(0.44, +0.1);
      complex(-3.39, +3.44), complex(-1.84, +5.53), complex(1.74, -0.04),
      complex(0, 0);
      complex(1.62, +3.68), complex(-2.77, -1.93), complex(0, 0),
      complex(0, 0)];
b = [complex(-8.86, -3.88), complex(-24.09, -5.27);
     complex(-15.57, -23.41), complex(-57.97, +8.14);
     complex(-7.63, +22.78), complex(19.09, -29.51);
     complex(-14.74, -2.4), complex(19.17, +21.33)];
[bOut, info] = f07vs(uplo, trans, diag, kd, ab, b)
```

```
bOut =
      0 + 2.0000i    1.0000 + 5.0000i
    1.0000 - 3.0000i   -7.0000 - 2.0000i
   -4.0000 - 5.0000i    3.0000 + 4.0000i
    2.0000 - 1.0000i   -6.0000 - 9.0000i
info =
      0
```